Backgammon Races

Introduction

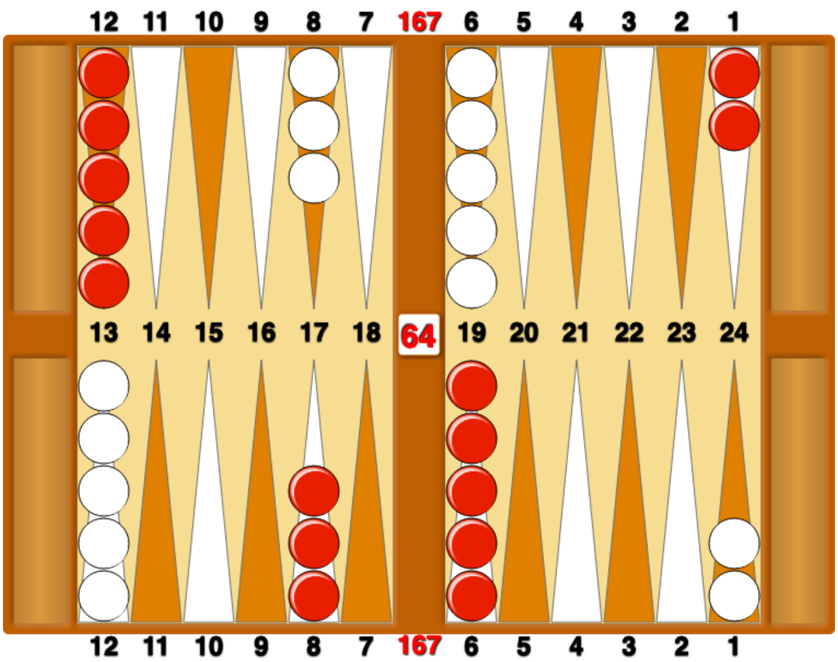
While we struggle to understand the complexity of the middle game of backgammon we continue to make progress in areas of the game where computers can assist us most easily. The obvious candidate area for analysis is the non-contact endgame where the race becomes of paramount importance. If we can produce and learn formulae to cover the majority of positions, then those who can remember and apply those formulae will have an edge over those who don't.

This short monograph summarises the development of formulae for racing situations. I do not claim to have derived any of these formulae but am summarising here the work of other backgammon theoreticians who have led us to where we are today. In particular I must acknowledge the work of Walter Trice whose work we will study in parts 3 and 4. I thoroughly recommend reading his book "Backgammon Boot Camp".

Part 1 – The Beginning

In this first part of the article I am going to look at the historical development of formulae for backgammon races. Until the 1970s there really weren't any formulae and the vast majority of backgammon players wouldn't have known a pip count if they'd met one! With the surge of interest that took place in the 1970s some order was finally imposed, and the first rudimentary formula based on the pip count appeared.

Just in case you don't know what a pip count is, it is the number of pips you must roll with the dice to bear off all your remaining checkers. The pip count in the starting position is 167 for each side.



The pip count is calculated by multiplying the number of checkers on each point by the value of the point and summing the total. In the diagram above Red's count is:

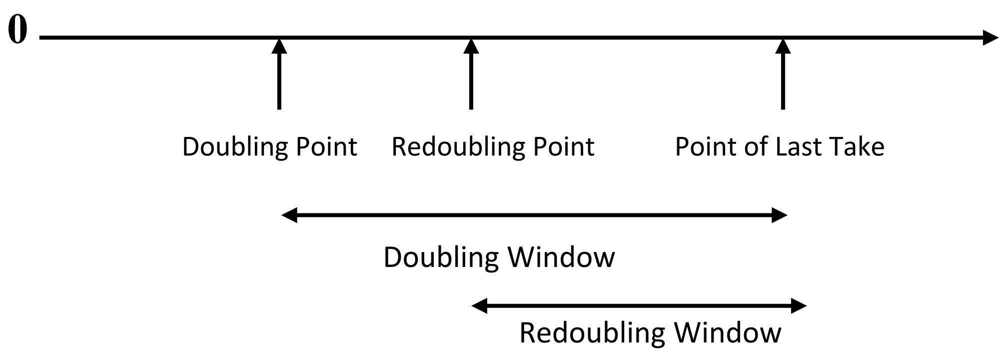
(2x24) + (5x13) + (3x8) + (5x6) = 48 + 65 + 24 + 30 = 167

We start by defining the two pip counts:

Leader's Pip Count (L): The number of pips required by the leader to bear off all his checkers.

Trailer's Pip Count (T): The number of pips required by the trailer to bear off all his checkers.

To help understand racing formulae a simple diagram is required. The line below shows the Trailer's pip count minus the Leader's pip count (T-L). Some of the key points on that line are highlighted:



Some further definitions:

Doubling Point: The leader's lead in the pip count is sufficient for him to offer an initial double.

Redoubling Point: The leader's lead in the pip count is sufficient for him to offer a redouble.

Point of Last Take: The point at which the difference in the pip counts is such that the trailer has a take but only just. Any increase in the difference would mean that the trailer must drop a double/redouble.

Doubling Window: The range of pip count difference within which double/take is the correct doubling decision.

Redoubling Window: The range of pip count difference within which redouble/take is the correct doubling decision.

The first formula that we are going to use to assist us in our decision making is simple but for all that it is reasonably effective and certainly a huge step forward from the visual inspection techniques that preceded it:

Let the leader's pip count be L.
Let the trailer's pip count be T.

If T is 8% greater than L then the Leader should double.
If T is 9% greater than L then the Leader should redouble.
If T is 12% greater than L then the Trailer should pass the double or redouble

This formula, while very basic, still works well for long races and, as noted above, it is a good guide for most races where the pip counts are greater than 50 (below that figure positional considerations normally but not always begin to exert a greater influence). For many years it was the only formula around and was used by all serious players.

If you want an even more basic method (very useful for those who absolutely hate mental arithmetic!) then Kit Woolsey has restated a variant of this formula in his book "The Backgammon Encyclopaedia Volume 1".

Quoting directly: "The main measure for a race is the pip count. Other things such as smoothness, crossovers, and gaps also play a part, but the pip count is usually the first thing to be looked at. The general theory is that for medium to long races, the doubling window centres around a 10% lead. Two pips more advantage for the leader and the trailer has a borderline take/pass. Two pips less for the leader and the leader has a borderline double (for an initial double he generally needs one fewer pip). This formula is pretty accurate for most races until the final bear-off stages are reached, assuming neither player has men buried on the lower points."

Part 2 – One Small Step for Man

As interest in the game grew and particularly as the understanding of the doubling cube improved so players came to realise that there must be ways to improve on this basic formula.

The problem is that there are other factors such as distribution, number of checkers etc. that need to be taken into account. To this end Edward O. Thorp (author of the blackjack book "Beat the Dealer") derived a more accurate formula. The Thorp formula goes as follows:

Compute the leader's adjusted pip count, L, as follows:

1. The leader's pip count.
2. Add two for each of the leader's remaining checkers.
3. Plus one for each checker on the 1 point.
4. Minus one for each point covered in the home board.
5. Plus 10% of the total so far, if that total is greater than 30.

Compute the trailer's adjusted pip count T, by following steps one to four, but omitting step five.

If T is greater than or equal to L-2, the leader should double.
If T is greater than or equal to L-1, the leader should redouble.
If T is greater than L+2, the trailer should pass.

Thorp's formula works well for many late game bear-offs but still fails to accurately assess some positions. This is because it doesn't make sufficient allowance for gaps or stacks.

Jeff Ward, a backgammon theoretician of the 1980's and author of the excellent "The Doubling Cube in Backgammon – Vol. 1", improved on Thorp's formula to create Ward's formula, the one still used by many of the top players today for basic races. Ward's formula goes as follows:

Compute the leader's adjusted pip count L, as follows:

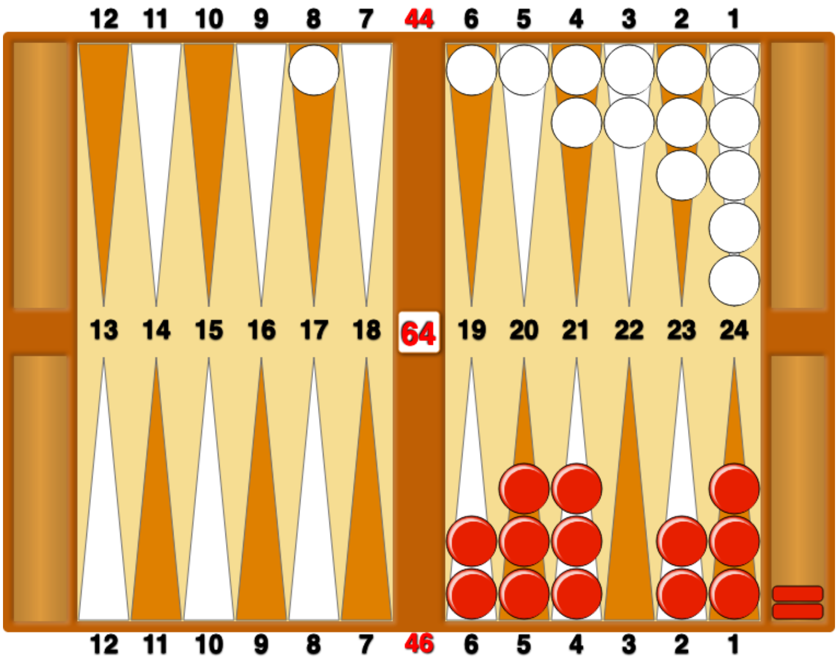
1. The leader's pip count.
2. Add two for each additional checker (over the number of checkers that the opponent has)
3. Plus two for each checker over two on the 1 point
4. Plus one for each checker over two on the 2 point.
5. Plus ½ for each checker outside the home board.
6. Plus 10% of the total so far, if that total is greater than 30.

Gaps are treated subjectively. A gap is an empty point in the home board when there is at least one checker on a higher numbered point. Gaps that can be filled by outfield checkers or internal gaps that will be filled easily are not penalised, e.g. an empty 3-point when there are lots of checkers on the 6-point, because 3's will fill the gap. Gaps that cannot be easily filled are penalised by subtracting one for each gap from the other player's adjusted pip count.

Compute the trailer's adjusted count T, in exactly the same way (except step 6).

If T is greater than or equal to L-2, the leader should double.
If T is greater than or equal to L-1, the leader should redouble.
If T is greater than L+2, the trailer should pass if the pip counts average 50 or less.
If T is greater than L+3, the trailer should pass if the pip counts average 75 or less.
If T is greater than L+4, the trailer should pass if the pip counts average 100 or less.

Here is Ward's formula in action (Red is on roll):



Red's pip count is 46.

Add 2 for the extra checker on the 1-pt. = 48
Add 10% = 52.8

White's pip count is 44.

Add 6 for the three extra checkers on the 1-pt = 50
Add 1 for the third checker on the 2-pt = 51
Add 4 for the two extra checkers = 55
Add ½ for the checker in the outer board = 55.5

White's adjusted count is 2.7 greater than Red's adjusted count so the correct cube action is double/redouble and drop as by Ward's criterion for a 50-pip race White's adjusted count would need to be within 2 pips of Red's. The basic formula and Thorp's formula both evaluate this position incorrectly.

This formula has been the bedrock for serious tournament players for over twenty years. However, it was produced in the years before computers were used to enhance our backgammon knowledge and whilst Ward tested it by doing manual rollouts it was not subjected to the scrutiny of computer analysis. How would it stand up against the vastly powerful computers of the early 2000s?

(It should be noted that the eminent backgammon theoretician Danny Kleinman also produced his own formula for races. However as this involved squaring numbers and not everybody is happy with having to do that I have omitted his work from this monograph.)

Part 3 – A Re-evaluation

The answer is that computers, coupled with the work of people such as Bill Robertie, Hugh Sconyers and especially Walter Trice, have increased our store of knowledge and refined our ability to make the right decisions in racing situations. We will now examine these improvements in some depth and end up with a set of formulae that represent the current thinking.

In fact, Ward was overoptimistic about the trailer's winning chances in longer races as long computer rollouts have shown. However, his method of adjusting the pip count has stood the test of time. Walter Trice has produced the following formula, known as **Formula 62**, which can be used for the vast majority of (relatively) straightforward races:

Perform the Ward Count as detailed above but omit step 6 for the leader, i.e. don't add 10% to the adjusted count. We then determine the trailer's point of last take as follows:

a) When the leader's adjusted pip count is 62 or less, subtract 5 and divide by 7, rounding down and then add the result to the leader's adjusted pip count. For example, for a pip count of 51 we subtract 5 to reach 46, divide by 7 to get 6.4 and round down to 6. Add 6 to the leader's adjusted pip count of 51. The point of last take is 57.
b) When the leader's adjusted pip count is higher than 62, divide by 10, round up and add 1 and then add the result to the leader's adjusted pip count. For example, if the leader's adjusted pip count is 76, 10% is 7.6, round up to 8 and add 1 giving 9. Add 9 to the leader's adjusted pip count. The point of last take is 85.
c) When the leader has an adjusted pip count of exactly 79 or 89 add an additional 1 to the point of last take.

The short form for **Formula 62** is:

-5, /7, down
/10, up, +1
switch at 62
for 79, 89 add 1

The leader's doubling and redoubling points can then be determined from the trailer's point of last take. The leader can redouble when the trailer is within two pips of his last take. Within three pips gives the leader an initial double. Example: If the leader's pip count is 67 then the trailer's point of last take is 75. The leader can redouble if the trailer has 73 or more and give an initial double with the trailer at 72.

One key point about racing theory that hitherto had not been appreciated and one that is certainly not intuitively obvious is that the *doubling window* – the gap between double or redouble and last take is constant, for all practical purposes, over a wide range of pip counts.

Part 4 – Wastage and Effective Pip Counts

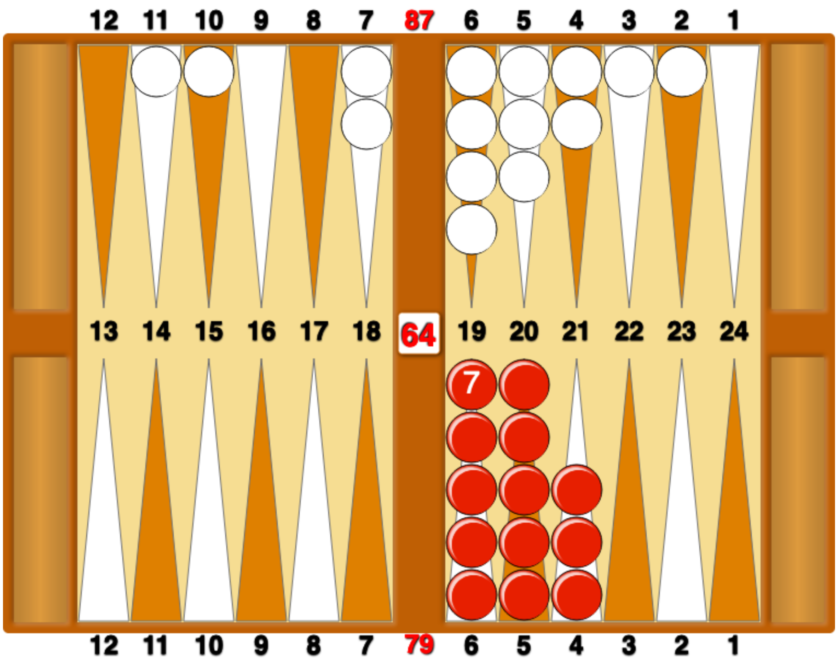
So far we have dealt with the fundamentals of racing but even the Ward formula plus Trice's formula 62 cannot cover all possible situations and we now need to delve even deeper in the mathematical properties of backgammon positions and associated bear-off problems so that we can solve problems where it is not a race between equals, i.e. one or other player has a position with some abnormal features such as a stack of checkers on one point.

Quite often in backgammon we reach positions where one player may have a "normal" distribution of checkers whilst his opponent has all his checkers stacked on the lower points in his home board. These are known as "Rolls vs. Pips" positions.

The first concept that we need to introduce is that of wastage. Every position has wastage. Take for example the simple situation where you have a checker on your 5-pt. If you roll a 6 to bear that checker off you have "wasted" one pip, i.e. you have rolled more than you need.

Wastage is the number of pips which you will roll on average to complete your bear off, minus your pip count.

Time to look at this position:



Red has achieved the optimal bear-off position, i.e. the one with least amount of wastage with all his checkers still on the board. He will waste on average 7.07 pips during the bear-off. It can be proved that there is no possible position with all fifteen checkers on the board that has less than 7.07 pips of wastage. This gives our first rule:

Rule 1: No position with all fifteen checkers has less than 7.07 pips of wastage

A second useful reference position is that of a closed board, i.e. two checkers on each of the home board points and the other three checkers already borne off.

Rule 2: The flat position wastes 10 pips.

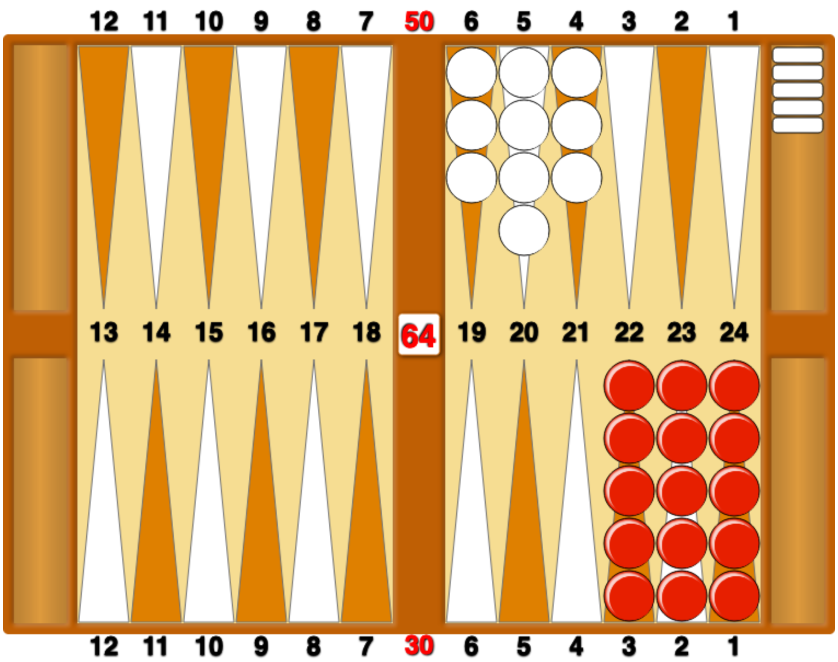
The second fundamental concept that we need to understand is **Effective Pip Count.**

Consider a position with 7 checkers on your 1-pt. You will bear these checkers off in 2, 3 or 4 rolls. Sparing you the calculations it will take you, on average, 3.5509259259 rolls. Not a very useful number. However, if you happen to multiply that by 8.1666667, which is the average dice roll in backgammon, you get 28.999.
If you do the same thing for a stack of 13 checkers on the 1-pt you get 50.000004. If you do the same exercise for all possible n roll (where n is the number of checkers remaining divided by 2 and rounded up) situations you get the sequence 8,15,22,29,36,43,50,57. This translates nicely into the formula **7n+1**. Both Walter Trice and Danny Kleinman should be given credit for this formula.

7n+1 gives you the Effective Pip Count (EPC) for stacked positions. In other words, if you rolled out a four-roll position (7 or 8 checkers) enough times and took note of the dice rolls, on average you would take 29 pips to bear off the checkers.

For other types of position the EPC is closer to the real pip count. The 3-5-7 position above (3 checkers on the 4-pt, 5 checkers on the 5-pt and 7 checkers on the 6-pt) has an actual pip count of 79 and 7.07 pips of wastage for an EPC of 86.07.

Some years ago Trice created the following position. It is known as **Proposition 57**:



This is an archetypal "rolls vs. pips" position (Red has the rolls, White has the pips).

Red starts with an EPC of close to 57 (from the 7n+1 formula). White starts with a real pip count of 50 plus 7 pips of wastage (from rule 1) for an EPC of about 57. Thus, the two sides are effectively equal and whoever wins the opening roll has the edge (unless White rolls 21).

Trice would offer to play either side of this position and would usually go home the winner. This is not because he was a great dice roller or could play the moves perfectly when playing the White side. It was because he could make optimal cube decisions when playing either side.

He developed the following rules of thumb for these "Rolls vs. Pips" positions:

Rule 3a: The point of last take for the trailer is when he is down a number of effective pips equal to the number of rolls to go minus three.

Rule 3b: The leader can double within two effective pips of last take.

Rule 3c: The leader can redouble within one effective pip of last take.

As an example, if the leader has a three-roll position (EPC of 22) then the trailer can take only if his EPC is also 22 or fewer. The leader can double when the trailer's EPC is 20 and redouble if it is 21.

Note that the doubling window is narrower than for pips vs. pips races. This is because there is less possibility for variance in the outcome when one side has a "rolls" position as only doubles make a significant difference - all other rolls are effectively equal.

There are certain positions that recur again and again in endgames and we are indebted once more to Walter Trice for deriving formulae for the EPC's in such positions. Learn these three rules – they will earn/save you a lot of money:

Rule 4: Fourteen checkers off, one checker somewhere outside the home board.

EPC = pip count + 4.7

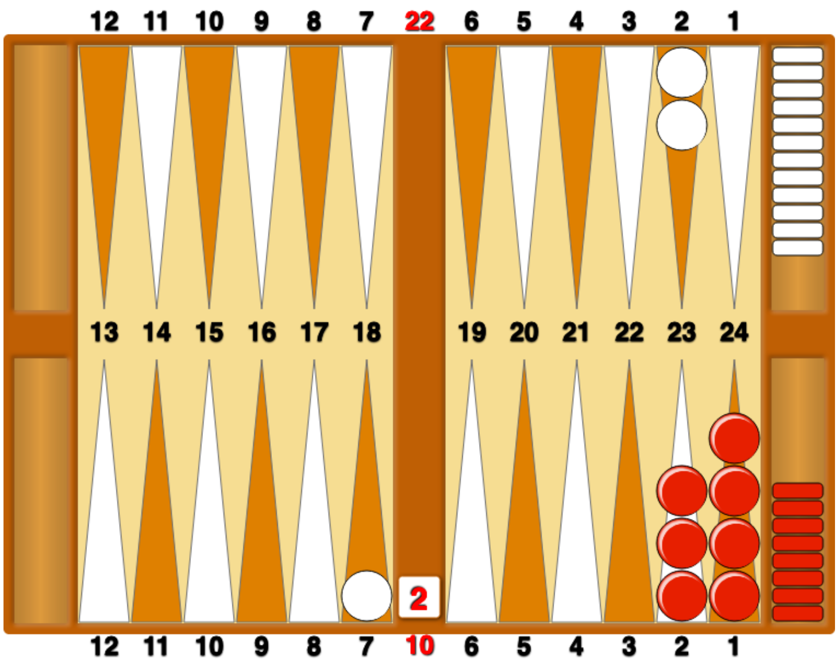
Rule 5: Thirteen checkers off, two outside checkers.

EPC = pip count + 5.2

Rule 6: Stack and straggler positions of the sort that happen when one player closes out his opponent, gets hit and then re-enters.

EPC = 3.5 x (total no. of checkers) + straggler (the pip count of the checker outside the home board)

Here's an example of a "stack and straggler" position that I had in my weekly chouette quite recently:



I was White and my opponent after long thought decided not to redouble. He promptly rolled double twos and cursed himself for not redoubling. If he'd had the stack and straggler formula it would have been easy.

He has an EPC of 29 (7x4 + 1). My effective pip count is (3 x 3.5) + 18 = 28.5

With a four-roll position my point of last take is 30 (see Rule 3a). He should redouble when my EPC is 29 (see Rule 3c). This position, therefore, is on the borderline for redoubling. The formula says no redouble (just). However, remember that formulae are only guidelines and when it's this close I would certainly redouble – I know a lot of people who would drop as White!

Summary

So, we have progressed to a point where, if we are prepared to do some work, we can be reasonably happy that we will get our doubling decisions correct more than 95% of the time for races and bear-offs and in time that figure will improve. For basic races we should use **Ward's Formula** plus **Trice's Formula 62**. For "Rolls vs. Pips" positions we should use **Trice's Six EPC Rules**.

Before Trice produced much of what we have learnt in parts 3, 4 and 5 of this monograph, he had a database on his computer with about 180 million positions and their associated equities. He wrote a program to test both the Thorp Formula and the Ward Formula against that database. Thorp got 153 million correct and Ward 163 million. When Thorp erred he was sometimes way off the mark, whilst Ward made only small errors.

With the refinements above I'm guessing that Ward plus Trice would now probably get 170 million or more correct which is a very reasonable percentage.

Hugh Sconyers has gone one further and produced a program that correctly calculates the exact equities and doubling decisions for all bear-off positions where both players have all their checkers in their home boards. The results can be found on a set of twelve CD's that Sconyers has produced and can be bought directly from him and other suppliers. The CDs are provided with a simple interface that lets the user look up any position he wishes to study. Note that nowadays the Extreme Gammon (XG) program will give you an accurate answer to virtually any bear-off problem so that The Sconyers CDs are largely reducndant.

Conclusion

Computers will refine our thinking over time and no doubt someone will produce a more accurate formula than those above, but we are fast approaching the point of diminishing returns and for all practical purposes the material in this monograph will suffice for the vast majority of players throughout their backgammon career.

Addendum (April 2021)

No sooner had I written that conclusion in 2013 than backgammon enthusiast Axel Reichert produced a short White Paper in 2014 entitled "Improved Cube Handling in Races: Insights with Isight".

The paper is quite technical and Isight refers to a piece of software that Axel uses in his work environment and which he used for his analysis of backgammon races. If you want to study his approach and reasoning, then you will need to read a copy of the White Paper which is forty pages long. What I will do here is present the key findings and recommendations so that you can use them in your own play.

Firstly, he redefined the calculation for Effective Pip Counts as follows:

<u>EPC</u>

Start with a straight pip count.

1. Add 5 pips.
2. Add 2 pips for each checker on the 1-pt.
3. Add 1 pip for each checker on the 2-pt.
4. Add 1 pip for each checker on the 3-pt.
5. Subtract 1 pip if the 1-pt and at least 3 other points are occupied.

<u>Cube Handling</u>

More importantly he developed a more sophisticated formula for cube handling as follows:

Start with a straight pip count for each player.

1. Add 1 pip for each additional checker on the board compared to the opponent.
2. Add 2 pips for each checker more than 2 on the 1-pt.
3. Add 1 pip for each checker more than 2 on the 2-pt.
4. Add 1 pip for each checker more than 3 on the 3-pt.
5. Add 1 pip for each empty space on the 4-pt, 5-pt and 6-pt (but only if the other player has one or more checkers on the corresponding point).
6. Add 1 pip for each additional crossover compared to the opponent.

With your adjusted pip count L and your lead ΔL (which can be negative),

$$P = 80 - L/3 + 2\Delta L$$

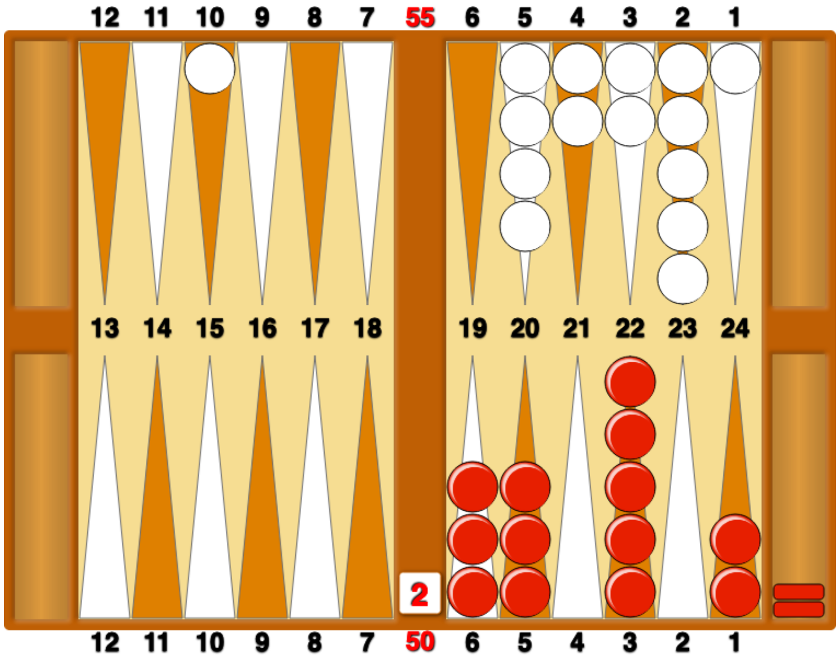
Where P is your approximated CPW (Cubeless Probability of Winning).

The cube action for money is:

P < 68	No double, take.
P = 68 or 69	Double, take.
P = 70 to 76	Redouble, take.
P > 76	Redouble, pass.

For matches you would need to adapt the (re)doubling window according to the match score. That goes well beyond the level of this short paper.

This method looks to be long and time-consuming but it is actually quite easy to apply after a little practice. I will give one example of the formula in practice. After that it is up to you to try it out on your own positions:



	Red	White
Pip Counts	50	55
Adjustments		
Checkers	0	2
Stack on 1	0	0
Stack on 2	0	3
Stack on 3	2	0
Gap on 4	1	0
Gap on 5	0	0
Gap on 6	0	1
Crossovers	0	1
Adjusted Counts	**53**	**62**

P = 80 – 53/3 + 9*2 = 80.33

As P is greater than 76 the correct cube action for this position is redouble and pass. Extreme Gammon agrees with this percentage to within a decimal point.

Bibliography

"The Doubling Cube in Backgammon – Vol 1" Dr. Jeff Ward

"Advanced Backgammon – Volume 2" Bill Robertie

"Backgammon Boot Camp" Walter Trice

To study Danny Kleinman's work in this area read his book:

"Vision Laughs at Counting with Advice to the Dicelorn – Volume 1"

Web Sites

www.back-gammon.info/sconyers/ Hugh Sconyers

To order the set of 12 CDs for exact bear-offs

www.chicagopoint.com/links.html General Links

The best general page for backgammon links

www.flintbg.com Carol Joy Cole

The best place to order backgammon books & materials (USA)

www.bgshop.com Chris Ternel

The best place to order backgammon books & materials (Europe)

http://www.bkgm.com Tom Keith

Backgammon Galore – more on races can be found here

Recommended Reading

There are many backgammon books available, albeit the vast majority of them only from specialist stockists such as Carol Joy Cole and Chris Ternel. What follows is my personal list of essential reading. Study these and you will truly become a much better player:

"Backgammon"	Paul Magriel
"Advanced Backgammon – Volumes 1 & 2"	Bill Robertie
"Modern Backgammon"	Bill Robertie
"Classic Backgammon Revisited"	Jeremy Bagai
"New Ideas in Backgammon"	Kit Woolsey & Hal Heinrich
"Boards, Blots and Double Shots"	Norm Wiggins
"The Backgammon Encyclopaedia – Volume 1"	Kit Woolsey
"The Backgammon Encyclopaedia – Volume 2"	Kit Woolsey
"How to Play Tournament Backgammon"	Kit Woolsey
"Backgammon Boot Camp"	Walter Trice
"The Doubling Cube in Backgammon – Volume 1"	Dr. Jeff Ward
"Vision Laughs at Counting with Advice to the Dicelorn"	Danny Kleinman
"The Backgammon Book" (extended version)	Oswald Jacoby & John Crawford
"Backgammon Praxis"	Marty Storer
"Backgammon to Win"	Chris Bray
"What Colour is the Wind?"	Chris Bray
"Second Wind"	Chris Bray
"Wind Assisted"	Chris Bray
"The Wind of Change"	Chris Bray
"Backgammon in the Wind"	Chris Bray
"What's Your Game Plan?"	Mary Hickey & Marty Storer
"Backgammon – from basics to badass"	Marc Brockman Olsen
"Conquering Backgammon"	Ed Rosenblum